In the presence of dielectrics these are sometimes more useful than the corresponding boundary conditions on **E** (Eqs. 2.31 and 2.23):

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\epsilon_0}\sigma, \tag{4.28}$$

and

$$\mathbf{E}^{\parallel}_{\text{above}} - \mathbf{E}^{\parallel}_{\text{below}} = 0. \tag{4.29}$$

You might try applying them, for example, to Probs. 4.16 and 4.17.

Problem 4.17 For the bar electret of Prob. 4.11, make three careful sketches: one of **P**, one of **E**, and one of **D**. Assume L is about $2a$. [*Hint:* **E** lines terminate on charges; **D** lines terminate on *free* charges.]

4.4 Linear Dielectrics

4.4.1 Susceptibility, Permittivity, Dielectric Constant

In Sects. 4.2 and 4.3 we did not commit ourselves as to the *cause* of **P**; we dealt only with the *effects* of polarization. From the qualitative discussion of Sect. 4.1, though, we know that the polarization of a dielectric ordinarily results from an electric field, which lines up the atomic or molecular dipoles. For many substances, in fact, the polarization is *proportional* to the field, provided **E** is not too strong:

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}. \tag{4.30}$$

The constant of proportionality, χ_e, is called the **electric susceptibility** of the medium (a factor of ϵ_0 has been extracted to make χ_e dimensionless). The value of χ_e depends on the microscopic structure of the substance in question (and also on external conditions such as temperature). I shall call materials that obey Eq. 4.30 **linear dielectrics.**[4]

Note that **E** in Eq. 4.30 is the *total* field; it may be due in part to free charges and in part to the polarization itself. If, for instance, we put a piece of dielectric into an external field $\mathbf{E}_0$, we cannot compute **P** directly from Eq. 4.30; the external field will polarize the material, and this polarization will produce its own field, which then contributes to the total field, and this in turn modifies the polarization, which ... Breaking out of this infinite regress is not always easy. You'll see some examples in a moment. The simplest approach is to begin with the *displacement,* at least in those cases where **D** can be deduced directly from the free charge distribution.

[4]In modern optical applications, especially, *non*linear materials have become increasingly important. For these there is a second term in the formula for **P** as a function of **E**—typically a *cubic* one. In general, Eq. 4.30 can be regarded as the first (nonzero) term in the Taylor expansion of **P** in powers of **E**.

In linear media we have

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_0 \mathbf{E} + \epsilon_0 \chi_e \mathbf{E} = \epsilon_0 (1 + \chi_e) \mathbf{E}, \tag{4.31}$$

so **D** is *also* proportional to **E**:

$$\mathbf{D} = \epsilon \mathbf{E}, \tag{4.32}$$

where

$$\epsilon \equiv \epsilon_0 (1 + \chi_e). \tag{4.33}$$

This new constant ϵ is called the **permittivity** of the material. (In vacuum, where there is no matter to polarize, the susceptibility is zero, and the permittivity is ϵ_0. That's why ϵ_0 is called the **permittivity of free space**. I dislike the term, for it suggests that the vacuum is just a special kind of linear dielectric, in which the permittivity happens to have the value $8.85 \times 10^{-12}\ \mathrm{C}^2/\mathrm{N}\cdot\mathrm{m}^2$.) If you remove a factor of ϵ_0, the remaining dimensionless quantity

$$\epsilon_r \equiv 1 + \chi_e = \frac{\epsilon}{\epsilon_0} \tag{4.34}$$

is called the **relative permittivity**, or **dielectric constant**, of the material. Dielectric constants for some common substances are listed in Table 4.2. Of course, the permittivity and the dielectric constant do not convey any information that was not already available in the susceptibility, nor is there anything essentially new in Eq. 4.32; the *physics* of linear dielectrics is all contained in Eq. 4.30.[5]

Material	Dielectric Constant	Material	Dielectric Constant
Vacuum	1	Benzene	2.28
Helium	1.000065	Diamond	5.7
Neon	1.00013	Salt	5.9
Hydrogen	1.00025	Silicon	11.8
Argon	1.00052	Methanol	33.0
Air (dry)	1.00054	Water	80.1
Nitrogen	1.00055	Ice (-30° C)	99
Water vapor (100° C)	1.00587	$KTaNbO_3$ (0° C)	34,000

Table 4.2 Dielectric Constants (unless otherwise specified, values given are for 1 atm, 20° C). *Source: Handbook of Chemistry and Physics,* 78th ed. (Boca Raton: CRC Press, Inc., 1997).

[5]As long as we are engaged in this orgy of unnecessary terminology and notation, I might as well mention that formulas for **D** in terms of **E** (Eq. 4.32, in the case of linear dielectrics) are called **constitutive relations**.